

## Instructions for GradeScope:

You may handwrite your answers or type them. Only type solutions if you are able to type the appropriate symbols. If **typed**, use at least a 12pt font, with **one question per page**. You can then directly submit the pdf file to GradeScope.

### If **handwritten**:

- Don't erase or cross out more than a word or two.
- Only write on one side of the page. Use dark, large, neat writing. Blank paper is better than lined paper.
- If you have a real scanner, great! Most people will scan their solutions using their phone. Use one of the recommended apps: For iOS, use Scannable by Evernote or Genius Scan. For Android, use Genius Scan. Do *not* just take regular photos.
- When scanning, have good lighting and try to avoid shadows.
- It is best to have **one question per scan**. If the solution is short, **fold the page in half** and scan just the half it is on, so there isn't so much blank space. Or, you can scan and then crop the scan. You don't need to scan parts (a), (b), etc. separately or put them on separate pages.
- It is best to upload a single pdf file rather than multiple image files. Most scanning apps will automatically combine your images into one pdf file.
- You must **check the quality** of your scans afterwards, and rescan if needed. Be sure to check the orientation of the scans and rotate if necessary.
- Make sure you are going to <https://gradescope.ca> not <https://gradescope.com>. You can access it through the course OWL site.
- You can **resubmit** your work any number of times until the deadline. If you are new to GradeScope, use this to practice your uploads.

See the GradeScope website for lots of information.

### Instructions for writing solutions:

- Homework is graded both on **correctness** and on **presentation**.
- Your proofs should be written in **complete sentences**, starting with capitals and ending with periods. They should be in **paragraph form**, similar to the proofs in the textbook. For example, you shouldn't put each sentence on a new line. Paragraphs should be used to break the proof into logical chunks.
- Don't use informal abbreviations such as three dots for "therefore." Logical symbols can be used when they clarify things to the reader, but English words can often be more effective.
- Include all of the steps that are needed to logically justify every claim you make. Do not include unnecessary steps. Try to be **concise and complete**.
- Do not submit any **rough work**.
- You should do the work **on your own**. Read the course syllabus for the rules about scholastic offences.

## Algebraic Topology, Homework Set 1

Due Saturday, January 22, by 11:59pm, using Gradescope. These questions use material up to and including the lecture on Thursday, January 13.

**Q1** (4 marks): Group the following spaces up to homotopy type, and within each group, up to homeomorphism. No proofs required!

$$\mathbb{R}^0, \mathbb{R}, \mathbb{R}^2, D^2, S^0, S^1, S^2, S^1 \times \mathbb{R}, S^2 \times \mathbb{R},$$

$$W = \{(x, y) \in \mathbb{R}^2 \mid x = 0 \text{ or } y = 0 \text{ or } x + y = 1\},$$

$$W' = \{(x, y) \in W \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\},$$

$$X_1 = \mathbb{R}^2 \setminus \{(0, 0)\},$$

$$X_2 = \mathbb{R}^2 \setminus \{(0, 0), (0, 1)\},$$

$$Y = D^2 \setminus \{(0, 0)\},$$

$$Z = \{(x, y) \in \mathbb{R}^2 \mid x \neq 0\}.$$

(Proofs required for the remaining questions.)

**Q2** (a) (3 marks): Show that  $S^1$  and  $X := S^1 \cup \{(x, 0) \in \mathbb{R}^2 \mid 1 \leq x \leq 2\}$  are homotopy equivalent.

(b) (1 mark): Is  $S^1$  a deformation retract of  $X$ ? Explain.

**Q3** (3 marks): Consider the map  $f : S^1 \rightarrow S^2$  sending  $(x, y)$  to  $(x, y, 0)$ . Show that  $f$  is null homotopic.

Recall that  $X$  is **contractible** if  $X$  is homotopy equivalent to a one-point space.

**Q4** (4 marks): Give an example of a contractible space  $X$  with a subspace  $A$  that is not contractible. Prove that  $X$  is contractible and  $A$  is not contractible. [Given what we have done so far, this limits the possible examples, but I sketched an argument in class that you can give the details for.]

**Q5** (3 marks): Show that a retract of a contractible space is contractible.